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An analytical method on circumferential periodic cracked pipes and shells

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Abstract

Based on the conservation law and bending theory, a very simple method to estimate stress intensity factors for circumferential periodic cracked pipes and shells is proposed. A series of closed form expressions of stress intensity factor are derived for cracked pipes and shells under different loads, such as bending, tension, three-point-bending, distributed load or combined loads. The examples show that the present method can produce the results obtained from complete shell analysis. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Pipes; Shells; Mechanics; Fracture; Stress intensity factor

1. Introduction

One of the tasks in fracture mechanics is to determine the stress intensity factors. For engineering applications and fracture theory, it is undoubtedly positive to determine stress intensity factors for complicated crack configuration using a simple method. To this goal, a G^* integral method (Xie et al., 1998a) was proposed to estimate stress intensity factors by bending theory and virtual work for two-dimensional cracks. However, most of cracks in engineering are three dimensional. Based upon numerical calculations some approximate formulae have been derived and compiled in many handbooks to provide a design tool for the practically working engineer. Unfortunately, use of these formulae often requires to stay within certain bounds concerning the conditions of loads and it is by no means easy or obvious to extend the limits of a given solution without performing a new cumbersome, numerical analysis of the problem.

For the two-dimensional elastostatic boundary value problems, the conservation law J_k have two components (Eshelby, 1951; Sih, 1969; Budiansky and Rice, 1973). If crack surface is parallel to axis x_1,

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the classical J integral is actually the first component J_1 and G^* integral (Xie et al., 1998a) is the second one J_2. They have the different physical meanings when apply them to solve crack problems. J integral means the crack extension energy release rate; and G^* integral the crack mouth widening energy release rate (Xie et al., 1998a). Two integrals yield the K_I using different components of strains and stresses for the same crack problem. To determine the K_I of cracked beams, for example, the strain and stress fields needed in the J integral method should be usually determined by the numerical analysis, but the strain and stress fields needed in G^* integral method can be given by the bending theory in mechanics of materials (Xie et al., 1998a). Some discussions (Xie, 1998) show that the G^* integral can be even used to determine the K_I of cracked pipes/shells within the framework of elementary mechanics.

It is the purpose of this paper to propose a method and technique to determine the stress intensity factors for circumferential periodic cracked pipes and shells. Derive the relationships between stress intensity factor and the three-dimensional G^* integral, and apply them to solve the periodic crack problems for cylindrical shells.

2. Circumferential periodic crack configuration

Consider m circumferential periodic cracks in the same cross section of a pipe or cylindrical shell. Cases $m = 3$, $m = 2$ and $m = 1$ are schematically shown in Fig. 1 as the examples. Following sections will give details about the analysis method proposed in present paper for circumferential periodic cracks.

3. Three-dimensional G^* integral

Consider a three-dimensional deformation field for which the displacement vector $\boldsymbol{u}$ depends on x_1, x_2 and x_3. From the conservation law, the three-dimensional G^* integral or crack surface widening energy release rate, similar to the two-dimensional one (Xie et al., 1998a), can be defined as

$$G^* = \int\int_{\Omega} \left(wn_2 - T_i u_{i,\,2} \right) \mathrm{d}\Omega \tag{1}$$

The Ω in Eq. (1) is one of the crack surfaces to which axis x_2 perpendicular, or a curved surface, boundary of which is the rim of the crack. These two surfaces construct a closed curved one. w is the strain energy density; T_i the stress vector acting on the outer side of Ω, n is the unit outward normal to Ω. Referring Figs. 3 and 4, the meaning of G^* integral is the energy release rate per unit translation of

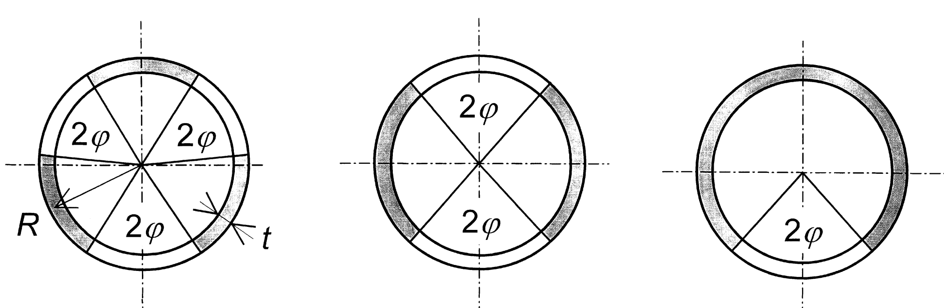

Fig. 1. Circumferential periodic crack configurations for cases $m = 3$, $m = 2$ and $m = 1$.

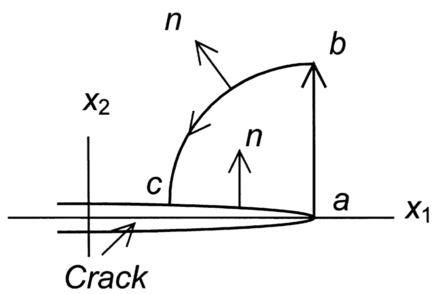

Fig. 2. Integration path next to crack tip region.

crack surface Ω_{af} in the x_2-direction or crack surface widening energy release rate when it is used to solve crack problems.

For two-dimensional elasto-static boundary value problem, G^* integral (Xie et al., 1998a) remains a similar form to Eq. (1). In this case, integration path is a curve in x_1, x_2 plane and can be expressed by symbol s.

A two-dimensional crack problem with unit thickness is shown in Fig. 2. Consider a path s_{abc} near to crack tip. The s_{ab} is a straight line and s_{bc} is a quarter of a circle. From the expressions of stress and strain in the field next to crack tip region, the following results had been obtained (Xie et al., 1998a)

$$G^* = \int_{s_{abc}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}s = \frac{\left(1-\mu^2\right)K_I^2}{2\pi E} \quad \text{(Plane strain)} \tag{2}$$

For a closed path $s = s_{abc} - s_{ac}$ in Fig. 2, from Eq. (2), it is not difficult to get (Xie et al., 1998a)

$$\oint_{s_{abca}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}s = \int_{s_{abc}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}s - \int_{s_{ca}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}s$$

$$= \frac{\left(1-\mu^2\right)K_I^2}{2\pi E} - \int_{s_{ca}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}s = 0 \tag{3}$$

then

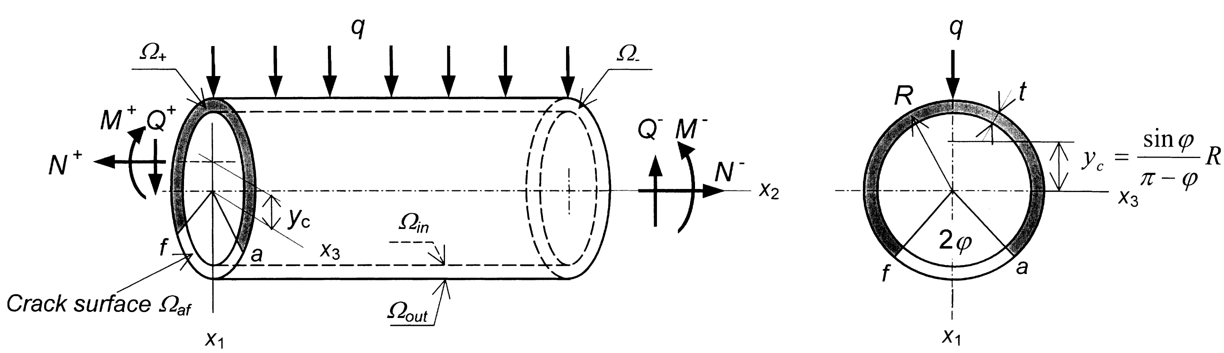

Fig. 3. Circumferential cracked pipe with one-half symmetry subjected to bending and tension for Mode I crack.

$$G^* = \int_{S_{ca}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}s = \frac{\left(1-\mu^2\right)K_I^2}{2\pi E} \tag{4}$$

4. Single edge cracked pipes (case $m = 1$)

A circumferential cracked pipe is shown in Fig. 3. The q is transverse load per unit length; Q is the transverse shear force ($Q^+ = 0$ for Mode I crack); M is the bending moment; N is the axial force and y_c the position of neutral axis in cracked cross section. The symbol '+' denotes the cracked cross section; '−' the remote uncracked cross section. A slender pipe possesses characteristics of both shells and bars. Making use of the concept of three-dimensional G^* integral for shells and bending theory in mechanics of materials for bar, the stress intensity factors of cracked pipe can be estimated.

4.1. Crack surface widening energy release rate based on G^* integral

Actually, the cracked pipe shown in Figs. 3 and 4 is a three-dimensional cylindrical shell. There are two singular plane strain fields discussed in above section next to the crack tip regions (Fig. 4). Let Ω_{acdf} denotes the right surface of crack in the shape of an arc. From Eqs. (1) and (4), the crack surface widening energy release rate can be given by

$$G^* = \int\int_{\Omega_{acdf}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}\Omega = 2t\int_{S_{ac}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}s + \int\int_{\Omega_{cd}} w\,\mathrm{d}\Omega$$

$$= \frac{t\left(1-\mu^2\right)K_I^2}{\pi E} + \int\int_{\Omega_{cd}} w\,\mathrm{d}\Omega \tag{5}$$

The physical meaning of Eq. (5) is energy release rate per unit translation of crack surface Ω_{acdf} in the x_2-direction or the crack surface widening energy release rate.

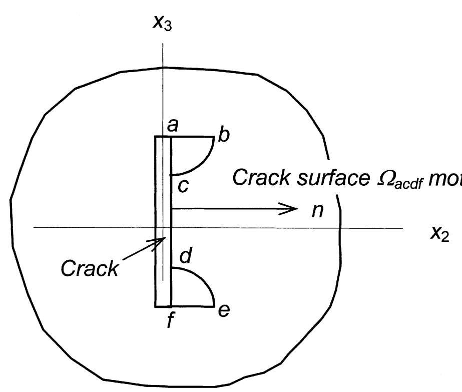

Fig. 4. Local wall of pipe around crack.

4.2. Crack surface widening energy release rate based on bending theory

When the crack in pipe is described by an elliptical hole under the condition of $b \to 0$ (Xie et al., 1998b) and the pipe shown in Fig. 3 and 5 is regarded as a bar, from bending theory, the strain energy is

$$U = \frac{N^2}{2EA}(l-b) + \int_0^b \frac{N^2}{2EA}\frac{\mathrm{d}x_2}{\bar{\gamma}_1(\varphi/\pi,\, x_2/b)} + \int_0^b \frac{\left(M^+ - Q^+x_2 - 0.5qx_2^2\right)^2}{2EI\bar{\gamma}_2(\varphi/\pi,\, x_2/b)}\mathrm{d}x_2 + \int_0^{l-b} \frac{\left(M^- + Q^-x_2 - 0.5qx_2^2\right)^2}{2EI}\,\mathrm{d}x_2 \tag{6}$$

$$A = 2\pi Rt,\ I = \pi R^3 t$$

where

$$\bar{\gamma}_1(\varphi/\pi,\, x_2/b) = 1 - \frac{\alpha}{\pi},\ \alpha = \varphi\sqrt{1 - \left(\frac{x_2}{b}\right)^2} \tag{7}$$

$$\bar{\gamma}_2(\varphi/\pi,\, x_2/b) = \frac{1}{\pi}\left[(\pi - \alpha) - \frac{1}{2}\sin 2\alpha - \frac{2\sin^2\alpha}{\pi - \alpha}\right] \tag{8}$$

Making use of Clapeyron's theorem, the work of the external loads, $V = 2U$; potential energy $\Pi = U - V$, then the crack surface widening energy release rate can also be given by the following equation from elementary theory

$$G^* = \lim_{b\to 0}\left(-\frac{\partial \Pi}{\partial b}\right) = \frac{N^2}{2EA}\left[\gamma_1\left(\frac{\varphi}{\pi}\right) - 1\right] + \frac{(M^+)^2}{2EI}\gamma_2\left(\frac{\varphi}{\pi}\right) - \frac{\left[M^- + Q^-l - 0.5ql^2\right]^2}{2EI} \tag{9}$$

where

$$\gamma_1\left(\frac{\varphi}{\pi}\right) = \int_0^1 \frac{\mathrm{d}t}{\bar{\gamma}_1(\varphi/\pi,\, t)},\ \gamma_2\left(\frac{\varphi}{\pi}\right) = \int_0^1 \frac{\mathrm{d}t}{\bar{\gamma}_2(\varphi/\pi,\, t)} \tag{10}$$

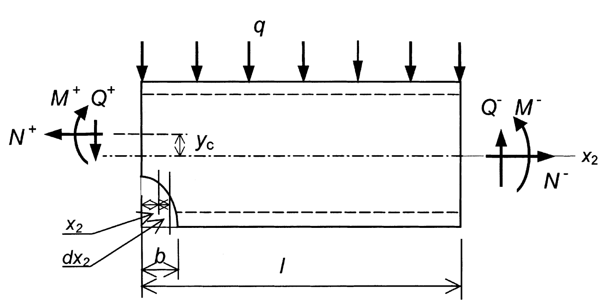

Fig. 5. An elliptical hole model for cracked pipes ($b \to 0$) (Xie et al. (1998b)).

$\gamma_1(\frac{\varphi}{\pi})$ and $\gamma_2(\frac{\varphi}{\pi})$ are respectively defined as the area-factor and inertia-factor at cracked cross section. From equilibrium conditions, Eq. (9) can be rearranged as

$$G^* = \frac{N^2}{2EA}\left[\gamma_1\left(\frac{\varphi}{\pi}\right) - 1\right] + \frac{(M^+)^2}{2EI}\gamma_2\left(\frac{\varphi}{\pi}\right) - \frac{\left[M^+ - NR\sin\varphi/(\pi-\varphi)\right]^2}{2EI} \tag{11}$$

4.3. Stress intensity factors

From Eqs. (5) and (11), following expression can be found.

$$\begin{aligned}&\frac{t(1-\mu^2)K_I^2}{\pi E} + \int\int_{\Omega_{cd}} w\,\mathrm{d}\Omega\\ &= \frac{N^2}{2EA}\left[\gamma_1\left(\frac{\varphi}{\pi}\right) - 1\right] + \frac{(M^+)^2}{2EI}\gamma_2\left(\frac{\varphi}{\pi}\right) - \frac{\left[M^+ - NR\sin\varphi/(\pi-\varphi)\right]^2}{2EI}\end{aligned} \tag{12}$$

Eq. (12) gives a relationship between stress intensity factor, loads acting on the cracked cross section, the area-factor and inertia-factor. Comparing with other terms in Eq. (12), $\int\int_{\Omega_{cd}} w\,\mathrm{d}\Omega$ is a small quantity because traction free on the crack surface. When neglecting this integral value, the Eq. (12) becomes

$$K_I = \left\{\frac{1}{4t^2R(1-\mu^2)}\left[N^2\left(\gamma_1\left(\frac{\varphi}{\pi}\right) - 1\right) + \frac{2(M^+)^2}{R^2}\gamma_2\left(\frac{\varphi}{\pi}\right) - \frac{2\left[M^+ - NR\sin\varphi/(\pi-\varphi)\right]^2}{R^2}\right]\right\}^{1/2} \tag{13}$$

Eq. (13) gives a formula to estimate stress intensity factors. In this formula, of the most interest is that stress intensity factors for cracked pipes and shells depend only on the axial force, bending moment in cracked cross section, area-factor and inertia-factor. Hence it is extremely simple and gives a closed form stress intensity factor for cracked pipes under different loads, such as bending, tension, three-point-bending, distributed load or combined loads. The Section 7 gives the examples.

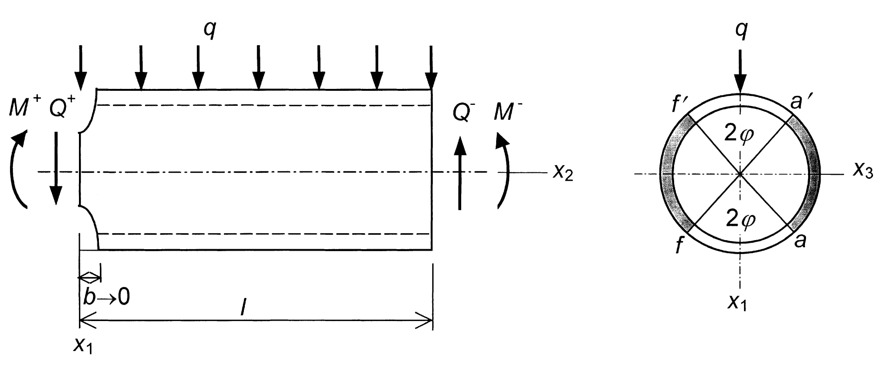

Fig. 6. Double edge cracked pipe for Mode I crack with one-half symmetry.

5. Double edge cracked pipes (case $m = 2$)

Fig. 6 shows a pipe with double edge cracks subjected to lateral loads. There are four singular plane strain fields next to the upper crack tip regions and the lower crack. For the lower crack, following expression can be given, similar to Eq. (5) (see Fig. 4 also)

$$G^*_{\text{lower}} = \iint_{\Omega_{acdf}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}\Omega = \frac{t\left(1-\mu^2\right)\left(K_I^2\right)_{\text{lower}}}{\pi E} + \iint_{\Omega_{cd}} w \, \mathrm{d}\Omega \tag{14}$$

For the upper crack

$$G^*_{\text{upper}} = \iint_{\Omega_{a'c'd\,'f\,'}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}\Omega = \frac{t\left(1-\mu^2\right)\left(K_I^2\right)_{\text{upper}}}{\pi E} + \iint_{\Omega_{c'd\,'}} w \, \mathrm{d}\Omega \tag{15}$$

Though tensile stress exists below the neutral axis and compressive stresses above, but the G^*-integrals in these four regions remain the same, i.e., $(K_I)_{\text{lower}} = -(K_I)_{\text{upper}} = K_I$. The crack surface widening energy release rate for the double edge cracked pipe is

$$G^* = G^*_{\text{lower}} + G^*_{\text{upper}} = \iint_{\Omega_{cddf}+\Omega_{c'd\,'d\,'f\,'}} \left(wn_2 - T_i u_{i,\,2}\right) \mathrm{d}\Omega = \frac{2t\left(1-\mu^2\right)K_I^2}{\pi E} + \iint_{\Omega_{cd}+\Omega_{c'd\,'}} w \, \mathrm{d}\Omega \tag{16}$$

Since the pipes can be regarded as a cracked slender member, the strain energy is

$$U = \int_0^{l-b} \frac{\left(M^- + Q^- x_2 - 0.5qx_2^2\right)^2}{2EI} \mathrm{d}x_2 + \int_0^b \frac{\left(M^+ - Q^+ x_2 - 0.5qx_2^2\right)^2}{2EI\bar{\gamma}_3(\varphi/\pi,\, x_2/b)} \mathrm{d}x_2 \tag{17}$$

where

$$\bar{\gamma}_3(\varphi/\pi,\, x_2/b) = 1 - 2\frac{\varphi}{\pi}\sqrt{1-\left(\frac{x_2}{b}\right)^2} - \frac{1}{\pi}\sin 2\varphi\sqrt{1-\left(\frac{x_2}{b}\right)^2} \tag{18}$$

From the bending theory of beams, the crack surface widening energy release rate is

$$G^* = \lim_{b\to 0}\left(-\frac{\partial \Pi}{\partial b}\right) = -\frac{\left(M^- + Q^- l - 0.5ql^2\right)^2}{2EI} + \frac{(M^+)^2}{2EI}\gamma_3\left(\frac{\varphi}{\pi}\right)$$

$$= \frac{(M^+)^2}{2EI}\left[\gamma_3\left(\frac{\varphi}{\pi}\right) - 1\right] \tag{19}$$

where

$$\gamma_3\left(\frac{\varphi}{\pi}\right) = \int_0^1 \frac{\mathrm{d}t}{\bar{\gamma}_3(\varphi/\pi,\, t)} \tag{20}$$

Let Eq. (16) is equal to Eq. (19), it yields

$$K_I = \frac{M^+}{\pi R^2 t}\sqrt{\pi R} f\left(\frac{\varphi}{\pi}\right) \tag{21}$$

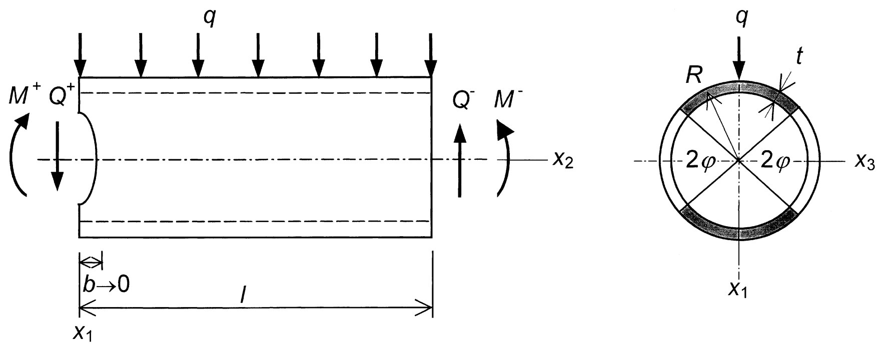

Fig. 7. Center-like cracked pipe for Mode I crack.

where the normalized stress intensity factor is

$$f\left(\frac{\varphi}{\pi}\right)=\frac{1}{2}\left\{\frac{\pi}{1-\mu^2}\left[\gamma_3\left(\frac{\varphi}{\pi}\right)-1\right]\right\}^{1/2} \tag{22}$$

Eq. (21), similar to Eq. (13), depends only on bending moment in cracked cross section and $\gamma_3(\varphi/\pi)$.

Another case of double edge-cracked pipe, center-like crack, is shown in Fig. 7. Each crack has a tensile singular stress field below neutral axis and a compressive above. By the same proceeding in Eqs. (14)–(19), following result can be obtained

$$\frac{2t(1-\mu^2)K_I^2}{\pi E}=\frac{(M^+)^2}{2EI}\left[\gamma_4\left(\frac{\varphi}{\pi}\right)-1\right] \tag{23}$$

where

$$\gamma_4\left(\frac{\varphi}{\pi}\right)=\int_0^1\frac{\mathrm{d}t}{1-(2\varphi/\pi)\sqrt{1-t^2}+(1/\pi)\sin\left(2\varphi\sqrt{1-t^2}\right)} \tag{24}$$

Then, the stress intensity factor is

$$K_I=\frac{M^+}{\pi R^2 t}\sqrt{\pi R}f\left(\frac{\varphi}{\pi}\right) \tag{25}$$

where the normalized stress intensity factor is

$$f\left(\frac{\varphi}{\pi}\right)=\frac{1}{2}\left\{\frac{\pi}{1-\mu^2}\left[\gamma_4\left(\frac{\varphi}{\pi}\right)-1\right]\right\}^{\frac{1}{2}} \tag{26}$$

6. Periodic cracked pipes (cases $m \geq 2$)

Circumferential periodic cracked pipe in tension is shown in Fig. 8. If there are m ($m \geq 2$) periodic cracks in a cross section, then $2m$ singular stress fields will exist in the regions near to crack tips. A similar procedure to above sections yields following expression

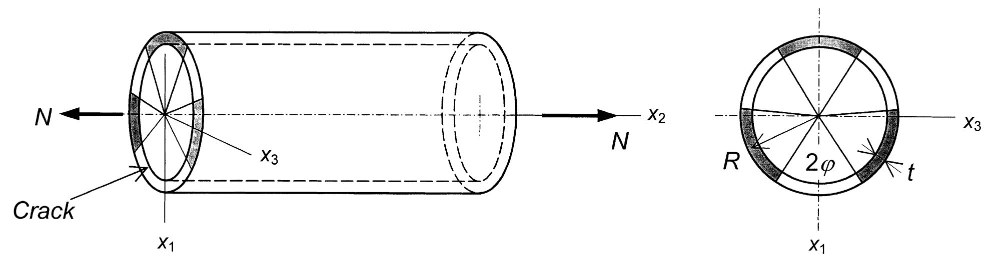

Fig. 8. Periodic cracked pipes with one-half symmetry subjected to tension for Mode I crack ($m = 3$ for schematically example).

$$m\frac{t(1-\mu^2)K_I^2}{\pi E} = \frac{N^2}{2EA}\left[\gamma_1\left(\frac{m\varphi}{\pi}\right) - 1\right], \quad m\varphi < \pi,\ m = 2, 3, 4, \ldots \tag{27}$$

where

$$\gamma_1\left(\frac{m\varphi}{\pi}\right) = \int_0^1 \frac{\mathrm{d}t}{1 - (m\varphi/\pi)\sqrt{1-t^2}} \tag{28}$$

then

$$K_I = \frac{N}{A}\sqrt{\pi R} f\left(\frac{m\varphi}{\pi}\right), \quad m\varphi < \pi,\ m = 2, 3, 4, \ldots \tag{29}$$

where the normalized stress intensity factor is

$$f\left(\frac{m\varphi}{\pi}\right) = \left\{\frac{\pi}{m(1-\mu^2)}\left[\gamma_1\left(\frac{m\varphi}{\pi}\right) - 1\right]\right\}^{1/2}, \quad m\varphi < \pi,\ m = 2, 3, 4, \ldots \tag{30}$$

When $m = 2$, Eq. (29) becomes an expression for double edge cracked pipe in tension.

7. Examples

7.1. Circumferential cracked pipe under bending

Consider a circumferential cracked pipe under bending shown in Fig. 9. From Eq. (13), the stress intensity factors is

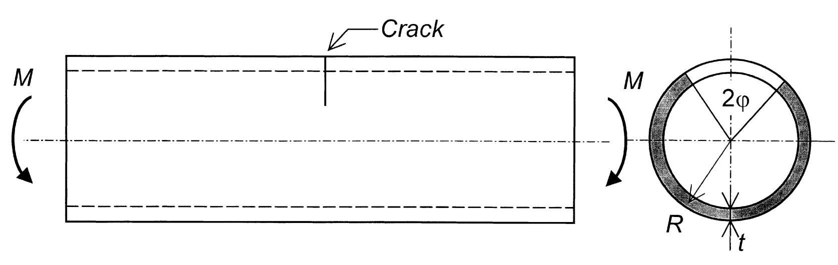

Fig. 9. Circumferential cracked pipe under bending.

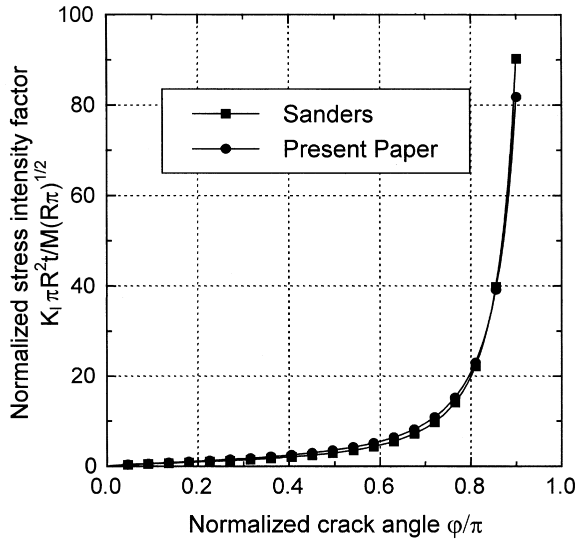

Fig. 10. Stress intensity factor for cracked pipe under tension. $t/R = 0.08$, $\mu = 0.3$.

$$K_I = \frac{M}{\pi R^2 t}\sqrt{\pi R} f\left(\frac{\varphi}{\pi}\right) \tag{31}$$

where normalized K_I is

$$f\left(\frac{\varphi}{\pi}\right) = \sqrt{\frac{\pi}{2(1-\mu^2)}\left(\gamma_2\left(\frac{\varphi}{\pi}\right) - 1\right)} \tag{32}$$

Fig. 10 compares the result calculated by Eq. (32) against the result from complete shell analysis of (Sanders, 1982, 1983).

7.2. Circumferential cracked pipe under tension

For circumferential cracked pipe under tension shown in Fig. 11, $M^+ = N \cdot y = N \cdot \frac{R \sin \varphi}{\pi - \varphi}$. Then Eq. (13) gives the following formula

$$K_I = \frac{N}{2\pi R t}\sqrt{\pi R} f\left(\frac{\varphi}{\pi}\right) \tag{33}$$

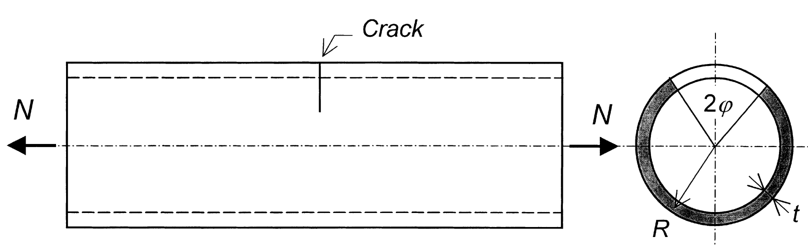

Fig. 11. Circumferential cracked pipe under tension.

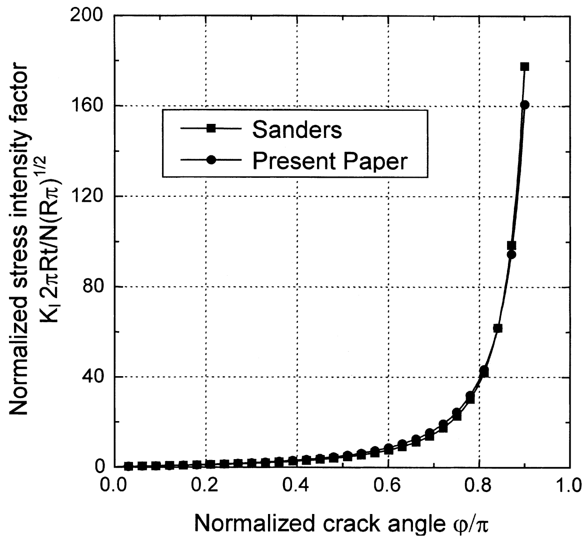

Fig. 12. Stress intensity factor for cracked pipe under tension. $t/R = 0.08$, $\mu = 0.3$.

where the normalized stress intensity factor is

$$f\left(\frac{\varphi}{\pi}\right) = \left\{\frac{\pi}{(1-\mu^2)}\left[\gamma_1\left(\frac{\varphi}{\pi}\right) + 2\left(\frac{\sin\varphi}{\pi-\varphi}\right)^2\gamma_2\left(\frac{\varphi}{\pi}\right) - 1\right]\right\}^{\frac{1}{2}} \tag{34}$$

Fig. 12 compares the result calculated by Eq. (34) against the result from complete shell analysis of (Sanders, 1982, 1983).

8. Theoretical discussions

For cracked pipes in above sections, two forms of crack surface widening energy release rate are derived from the concept of three-dimensional G^* integral and elementary mechanics. The former gives a function of K_I; the later yields a function of loads. Finally, the relationship between stress intensity factors and loads for cracked pipes is given.

Alternatively, this relationship for cracked pipes can also be derived directly from three-dimensional conservation law. Consider a closed curved surface $\Omega = \Omega_+ + \Omega_{acdf} + \Omega_{\text{in}} + \Omega_{\text{out}} + \Omega_-$ for the cracked pipes shown in Figs. 3 and 4 for example. The Ω_- denotes the area of remote uncracked cross section. The Ω_{in} and Ω_{out} are the inner surface and outer surface of pipe wall respectively, and the inner surface is action free; Ω_+ the cross section of crack ligament. Along these surfaces, integral values are respectively

$$\iint_{\Omega_{\text{in}}} (wn_2 - T_i u_{i,2})\,\mathrm{d}\Omega = 0 \tag{35}$$

$$\iint_{\Omega_{\text{out}}} (wn_2 - T_i u_{i,2})\,\mathrm{d}\Omega = q(\tilde{u}_1^+ - \tilde{u}_1^-) \tag{36}$$

$$\iint_{\Omega_-}\left(wn_2 - T_i u_{i,\,2}\right)\,\mathrm{d}\Omega = \bar{w}^- - Q^-\tilde{u}^-_{1,\,2} - N\tilde{u}^-_{2,\,2} - M^-\psi'^{\,-} \tag{37}$$

$$\iint_{\Omega_+}\left(wn_2 - T_i u_{i,\,2}\right)\,\mathrm{d}\Omega = -\left(\bar{w}^+ - Q^+\tilde{u}^+_{1,\,2} - N\tilde{u}^+_{2,\,2} - M^+\psi'^{\,+}\right) \tag{38}$$

The $\tilde{u}_i$ shows the displacement of neutral axis, which is determined by bending theory in mechanics of materials.ψ is the cross section rotation and $\bar{w}$ the strain energy density per unit length of pipe.

Along the closed curved surface $\Omega = \Omega_+ + \Omega_{acdf} + \Omega_{\mathrm{in}} + \Omega_{\mathrm{out}} + \Omega_-$, the following expression can be given from conservation law $\oiint_{\Omega}(wn_2 - T_i u_{i,\,2})\,\mathrm{d}\Omega = 0$ and Eqs. (5), (35)–(38), noting that n is outward normal to Ω.

$$\frac{t\left(1-\mu^2\right)K_I^2}{\pi E} + \iint_{\Omega_{cd}} w\,\mathrm{d}\Omega = B^- - B^+ \tag{39}$$

where

$$B^- = \bar{w}^- - Q^-\tilde{u}^-_{1,\,2} - N\tilde{u}^-_{2,\,2} - M^-\psi' - q\tilde{u}^-_1 \tag{40a}$$

$$B^+ = \bar{w}^+ - Q^+\tilde{u}^+_{1,\,2} - N\tilde{u}^+_{2,\,2} - M^+\psi' - q\tilde{u}_1 \tag{40b}$$

and then

$$K_I = \left\{\frac{\pi E}{t\left(1-\mu^2\right)}\left(B^- - B^+\right)\right\}^{1/2} \tag{41}$$

Eq. (39) is equivalent to Eq. (12) and Eq. (41) to (13). Further simple derivation to Eq. (41) will yield (13). By proceeding in this manner, Eqs. (21), (25) and (29) can also be obtained from the three-dimensional conversation law.

9. Conclusions

Based on the three-dimensional G^* integral and bending theory, a theory and method of determining stress intensity factors for circumferential periodic cracked pipes and shells are established. The examples indicate that this method is valid and convenient to engineering applications. With the series expansion of Eqs. (10), (20) and (24), the proposed method will become simpler.

These solutions in present paper is the primary step to calculate the crack opening area; this, in turn, provides a means to evaluate the leak rate of pressurized pipes in nuclear power engineering. Paper concerning these topics is in preparation.

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